Appendix F
Game Theory

This appendix provides some elementary facts about game theory and equilibrium concepts and should serve as a refresher for those readers with some background in game theory. However, for a proper and more complete explanation of the theory, the reader should refer to Fudenberg and Tirole [195], Myerson [399] or Mas-Colell, Whinston, and Green [365] (from which the material in this appendix is obtained). Also, Tirole [513] provides a User's Manual on Game Theory.

The Normal Form of Games

A game consists of a set of *players,* the *actions* that they can take (or in other words, the rules of the game), and the *information* that each player possesses at the time he takes his action. For each possible set of actions, the game defines a set of *outcomes* and *payoffs* for each player (such as how much profit or utility each player gets).

For instance, in the Bertrand pricing game (Section 8.4.1.4), there are n players (firms in the oligopoly). Their action space is the prices they set. Each possesses information that all the demand goes to the lowest-priced firm, and all have the same marginal costs. The outcome is that the demand goes to the lowest-priced firm. The payoffs are the revenues minus costs (profits).

Formally, let there be n players, let $\mathcal{H}_i$ be the collection of player i's information sets and $C(H) \subset \mathcal{A}$ be the set of actions possible for player i with information set *H.* A *(pure) strategy* for player i is a function $s_i : \mathcal{H}_i \to \mathcal{A}$,—that is, the player has a mapping from each possible information set to a unique action. Moreover, the actions have to be feasible, so we assume that the strategy map is such that $s_i(H) \in C(H)$ for all $H \in \mathcal{H}_i$. Each player, given a set of pure strategies, can also randomize over these strategies (his strategy is to choose one of his pure strategies with a certain probability). This creates what are called *mixed strategies.*

A game's actions, outcomes, and payoffs can be defined by an extensive form or a normal form. Here we concentrate on the normal form. The normal form of the game is a specification of a set of possible strategies S_i for player i, and a payoff function $u_i(s_1, \ldots, s_N)$ if each player plays strategy $s_i \in S_i, i = 1, \ldots, N$. The game Γ is defined as the triple $\Gamma = [N, \{S_i\}, \{u_i(\cdot)\}]$.